Creditreform Rating AG

Technical Documentation

Portfolio Loss Distributions

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Creditreform Rating

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Introduction

Ratings in the context of „Institutional Investor Debt“ and „Structured Finance“ utilize quantitative methods which support and extend the qualitative analyses during the rating process. This document describes the use of a simulation technique to determine portfolio losses and corresponding loss distributions. The document supplements the publicly available rating methodologies related to „Institutional Investor Debt“ and „Structured Finance“ issuances with a technical documentation and the key rating assumptions which define the statistical-mathematical foundation and the simulation approach used by CRA to derive portfolio losses and their distribution.

This enables all parties involved, investors and the public to understand a rating of CRA. This document will be updated regularly to reflect changes in the methodology. All rating methodologies and this technical documentation are available on CRA´s homepage at www.creditreform-rating.de.

The processes which will be described in the following are (1) the time-dependent representation of probabilities of default and (2) the statistical derivation of (one- or multi-period) loss distributions of a portfolio of credits. The calculation of all parameters of interest is based on a stochastic financial model commonly known as the „one-factor model“, a special case of structural models which goes back to the work of Robert Merton.[1]

1 The One-Factor Model

The one-factor model assumes that the value of a company is dependent on the realization of a latent variable which is modelled as a stochastic process. With a finite time horizon, this reduces to assuming that each of *n* credits in the portfolio with $i \in \{1, \ldots, n\}$ can be modelled by a latent variable R_i. The value of the latent variable depends on a systematic factor which is common to all credits in the portfolio, and an idiosyncratic factor that is entity-specific. Let *Y* be a systematic factor and ε_i idiosyncratic factors with $\forall i \in \{1, \ldots, n\}$. Then it is possible to define a latent variable $\forall i \in \{1, \ldots, n\}$ such that

$$R_i = \sqrt{\varrho_i} \cdot Y + \sqrt{1 - \varrho_i} \cdot \varepsilon_i$$

In particular, it is assumed that $\varepsilon_1, \ldots, \varepsilon_n \sim N(0,1)\ i.i.d.$, that is all factors are „independently and identically distributed“. One can show that $\sqrt{\varrho_i}$ is the correlation of the latent variable R_i und Y, and therefore a measure of the dependence between the latent variable and the systematic factor. This can be shown as follows:

[1] An overview to the theoretical background and the statistical-mathematical basis of relevant models can be found in:

- Bluhm, Overbeck (2007): *Structured Credit Portfolio Analysis, Baskets & CDOs*. Chapman & Hall/CRC. London.
- Trueck, Rachev (2009): *Rating Based Modeling of Credit Risk. Theory and Application of Migration Matrices*. Elsevier. Amsterdam.

$$Corr(R_i, Y) = \frac{Cov(R_i, Y)}{\sigma(R_i) \cdot \sigma(Y)} = Cov(R_i, Y) = E[(R_i - E[R_i])(Y - E[Y])] = E[R_i Y] = \cdots = \sqrt{\varrho_i}$$

Since R_i is a linear combination of random variables which are $\sim N(0,1)$ distributed, the distributional assumptions are valid for R_i as well, which can be shown by calculating its expected value and variance.

1.1 Asset Correlation

Dependencies between the asset values of different credits $i \neq j; i, j \in \{1, \ldots, n\}$ are important and can be derived from the dependencies of single entities with the systematic factor. The dependency of R_i and R_j with the systematic factor implies a correlation $Corr(R_i, R_j)$ between the assets. Since R_i and R_j are distributed standard normal, this correlation also displays the model´s entire correlation structure. In particular, it is true that:

$$Corr(R_i, R_j) = \frac{Cov(R_i, R_j)}{\sigma(R_i) \cdot \sigma(R_j)} = Cov(R_i, R_j) = E[(R_i - E[R_i])(R_j - E[R_j])] = E[R_i R_j] = \sqrt{\varrho_i \varrho_j}$$

CRA assumes a default pair-wise asset correlation of 10% when analysing credit portfolios and may adjust this assumption based on particular concentrations (i.e. industry, geography etc.) and following a decision of the rating committee.

1.2 Probability of Default

In relation to $R_i = \sqrt{\delta_i} \cdot Y + \sqrt{1 - \delta_i} \cdot \varepsilon_i$, a default event can be defined to occur when R_i is below a critical threshold c_i. Defining the PD for all credits in the portfolio $i \in \{1, \ldots, n\}$ as p_i,

$$p_i = P(R_i < c_i)$$

Since the model is first analysed on a fixed time-horizon, every realisation of the latent random variable R_i will indicate whether or not a default has occured within the time-horizon and when it occurred. Since the distribution function Φ of the standard normal distribution is continuous:

$$p_i = P(R_i \leq c_i) \Rightarrow c_i = \Phi^{-1}(p_i)$$

The modelling assumptions of the latent variable then allow for a derivation of the distribution of losses. Let $D_i = \{R_i < c_i\}$ define an event in which the latent variable falls below the critical threshold c_i and the credit *i* experiences a default in the corresponding fix time horizon. Then it is possible to define an indicator variable to display the state of the credit („default / no default") at any point. This is a binary variable; its values can be interpreted as the outcome of a Bernoulli experiment with distributional parameters

$$1_{D_i} = \begin{cases} 1, & R_i < c_i \\ 0, & R_i \geq c_i \end{cases}$$

$P(1_{D_i} = 1) = P(R_i < c_i) = p_i$ measures the probability of success of the Bernoulli experiment.

As the valuation of portfolio credits entails the use of stress scenarios, it is important to know the conditional probability of default of a credit *i* with respect to different realisations of the systematic factor, which is often interpreted as a representation of macroeconomic variables (i.e. GDP). The conditional probability of default can be calculated as:

$$p_i(y) = P(R_i < c_i | Y = y) = P\left(\sqrt{\varrho_i} \cdot Y + \sqrt{1-\varrho_i} \cdot \varepsilon_i < c_i \middle| Y = y\right) = P\left(\varepsilon_i < \frac{c_i - \sqrt{\varrho_i} \cdot Y}{\sqrt{1-\varrho_i}} \middle| Y = y\right)$$

$$= \Phi\left(\frac{c_i - \sqrt{\varrho_i} \cdot y}{\sqrt{1-\varrho_i}}\right) = \Phi\left(\frac{\Phi^{-1}(p_i) - \sqrt{\varrho_i} \cdot y}{\sqrt{1-\varrho_i}}\right)$$

2 Time-Dependent Probabilities of Default

Up to this point, the time-horizon has been held fixed and probabilities of default have implicitly been derived for a horizon of one period. This section describes methods to derive time-dependent probabilities of default (“Credit Curves” or “PD Term Structure”). The basis for the calculation of time-dependent probabilities of default is CRA´s migration matrix of rating classes. Typically, it is calculated on a time-horizon of one year. To extend these to multiple-period time horizons, CRA uses a time-homogenous Markov-chain approach and a generator matrix to derive cumulative probabilities of default:

Figure 1: Example PD Term Structure | Source: CRA

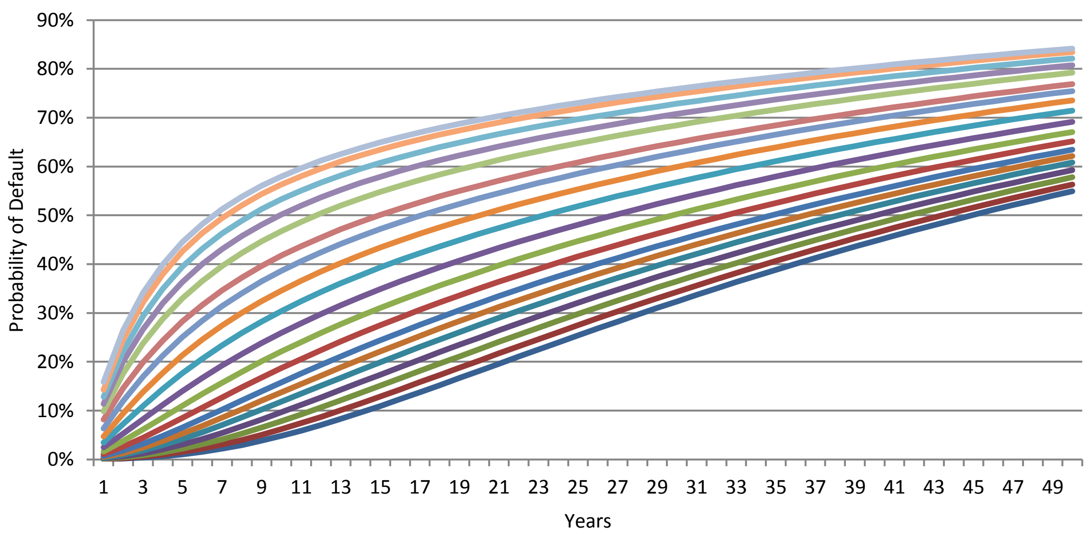
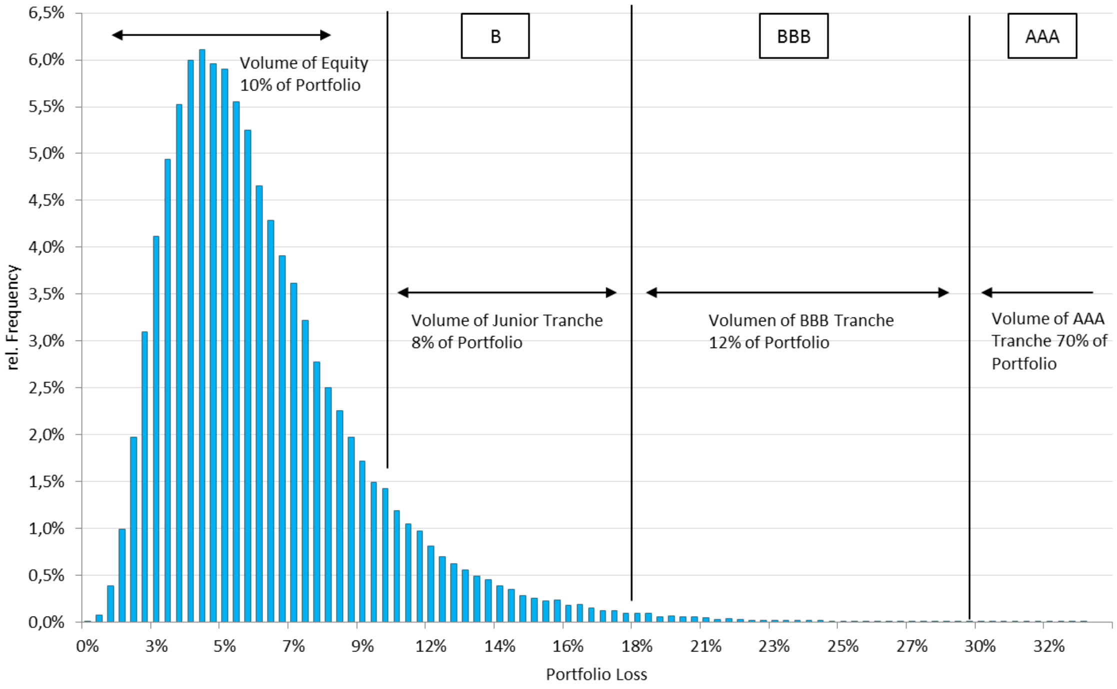


Figure 1 presents a hypothetical PD Term Structure for different rating classes. Appendix 1 presents an idealized version of the current PD Term Structure used by CRA, which has been calibrated on the basis of CRA internal proprietary default data.

3 The Loss Distribution

The loss distribution is the probability distribution of portfolio losses; it corresponds to the sum of defaulted nominal amounts taking into account the particular loss severities and recoveries. Any incomplete and non-timely payment of an obligation of payment will be defined as a default.

Figure 2: Example of a portfolio loss distribution and relevant risk measures | Source: CRA



The parameters which define the loss distribution are single address probabilities of default, exposure at default (EAD), loss given default (LGD) and asset- or default correlations. Exposure is commonly defined as any amount that is at risk of a defult at a certain point in time.

To derive the loss distribution, CRA uses the realizations of a Monte-Carlo simulation. In doing so, the magnitude of losses (of one or more portfolio credits) in the event of default will also be accounted for. The asset- or default correlation will be included because the joint probability of default is described as in 1.2. above. In particular, it is true that $P(default) > (Pdefault\ obligor\ 1) \cdot P(default\ obligor\ 2)$ and the probability of a default is larger in comparison to a case of independence.

The loss distribution can be derived as the cumulative frequency distribution of portfolio losses. It displays the probability x% of losses exceeding a certain value, and can be used to calculate Value-at-Risk. In addition, single tranches can be rated once the respective transaction capital structure is known.

4 Appendix

Table 1: Idealized PD-Term Structure (June 2018) | Source: CRA

Class / Years, PD	AAA	AA	A	BBB	BB	B	CCC	C-CC
1	0.0%	0.0%	0.1%	0.3%	1.3%	5.2%	14.9%	29.1%
2	0.0%	0.1%	0.2%	0.6%	2.6%	9.4%	24.8%	43.6%
3	0.0%	0.1%	0.4%	1.1%	3.9%	12.9%	31.4%	51.6%
4	0.0%	0.3%	0.7%	1.6%	5.1%	15.8%	36.0%	56.4%
5	0.1%	0.4%	1.1%	2.1%	6.4%	18.2%	39.3%	59.5%
6	0.1%	0.6%	1.5%	2.7%	7.6%	20.3%	41.7%	61.7%
7	0.2%	0.8%	2.0%	3.3%	8.8%	22.1%	43.6%	63.2%
8	0.3%	1.1%	2.5%	4.0%	9.9%	23.7%	45.2%	64.4%
9	0.4%	1.4%	3.1%	4.7%	11.0%	25.2%	46.5%	65.4%
10	0.5%	1.7%	3.7%	5.5%	12.1%	26.5%	47.6%	66.2%
11	0.6%	2.1%	4.4%	6.2%	13.1%	27.7%	48.6%	66.9%
12	0.8%	2.5%	5.1%	7.0%	14.1%	28.8%	49.4%	67.5%
13	1.0%	2.9%	5.8%	7.8%	15.1%	29.8%	50.2%	68.1%
14	1.2%	3.3%	6.5%	8.6%	16.1%	30.8%	51.0%	68.6%
15	1.5%	3.8%	7.3%	9.4%	17.0%	31.8%	51.7%	69.0%
16	1.8%	4.3%	8.0%	10.2%	17.9%	32.7%	52.3%	69.5%
17	2.1%	4.9%	8.8%	11.0%	18.8%	33.5%	53.0%	69.9%
18	2.4%	5.4%	9.6%	11.9%	19.7%	34.3%	53.6%	70.3%
19	2.8%	6.0%	10.4%	12.7%	20.6%	35.1%	54.1%	70.7%
20	3.2%	6.6%	11.2%	13.5%	21.4%	35.9%	54.7%	71.0%

Figure 3: Idealized PD-Term Structure, graphical display (June 2018) | Source: CRA