

Creditreform Rating AG Supplementary Methodology

# Derivation of Interest-Rate Scenarios

DRAFT

Neuss, May 19, 2026  
Version 1.0 DRAFT

**Creditreform**   
**Rating**

## Table of Content

<b>1</b>	<b>INTRODUCTION .....</b>	<b>3</b>
<b>2</b>	<b>THE COX-INGERSOLL-ROSS MODEL OF INTEREST RATES.....</b>	<b>4</b>
<b>3</b>	<b>CIR MODEL PARAMETERIZATION .....</b>	<b>5</b>
<b>4</b>	<b>INTEGRATION OF SIMULATED RATES INTO CASH FLOW MODELS .....</b>	<b>7</b>

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## 1 Introduction

This document describes the framework Creditreform Rating AG (henceforth, CRA) employs to analyze the impact of changes in interest rates on transactions that we rate, including, but not limited to structured-finance securities as well as structured credit (asset-based financing). The scope does not extend to transactions without interest-rate risk, to transactions in which CRA considers such risks to be either comprehensively mitigated (e.g., through appropriate hedging), immaterial, or to cases assessed using other approaches. CRA will conduct regular reviews of this article and will publish an update in the event of any changes to our approach. CRA's rating methodologies and code of conduct are freely available on our web site ([www.creditreform-rating.de](http://www.creditreform-rating.de)).

Securitization transactions may be susceptible to changes in interest rates for several reasons. In certain cases, a securitization vehicle may acquire debt which is based on floating rates and refinance the purchase through the issuance of fixed-income securities, giving rise to an asset-liability mismatch. Alternatively, in context of a structured finance transaction, a securitization vehicle purchases fixed-rate mortgages which may have to be sold prior to maturity, hence exposing the securitization vehicle to a valuation risk with respect to these mortgages. Interest rate risk can also result from structures with revolving, short-term liabilities that necessitate the ongoing refinancing of longer-term assets during the transaction's term. Absent mitigating factors, such borrowers are exposed to refinancing risk. Finally, a shift in rates may affect excess spread dynamics, potentially leading to the breach of performance-based triggers, which ultimately might impact performance (particularly the one of subordinated tranches in structured-finance transactions).

The framework aims to produce forward-looking scenarios for key reference rates, such as the three-month EURIBOR, that reflects the potential impact of interest-rate volatility on transaction performance. These scenarios are generated by means of a simulation-based approach. The rate trajectories outputted by the simulation-based model are subsequently integrated into the quantitative cash flow model corresponding to a rated transaction's structure. The rate trajectories hence serve as a key input for assessing the structure's sensitivity to changes in applicable reference rates.<sup>1</sup>

Section 2 outlines the theoretical framework CRA employs to simulate interest rate dynamics. In section 3, we discuss the parameterization of the model. Section 4 describes how the interest rate framework is integrated into CRA's cash flow models, both for large-homogeneous portfolio structures

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<sup>1</sup> Besides the simulated rates, other, transaction-specific features such as the presence or absence of interest-rate swaps are highly relevant to the analysis. However, this document focuses solely on the creation of interest-rate scenarios. Please cf. our product-type specific methodologies which lay out other aspects relevant to CRA's assessment of interest-rate risks.

commonly found in structured finance as well as for small-portfolio structures typical of structured credit (asset-based financing). The section also includes illustrative model output.

## 2 The Cox-Ingersoll-Ross Model of Interest Rates

In order to simulate interest rates, we rely on a variant of the Cox, Ingersoll, and Ross (1985) model (henceforth, CIR model).<sup>2</sup> The variant of the CIR process we use is popular with practitioners both for its tractability (it is a single-equation model) as well as for its wide applicability to all kinds of interest rates. The broad applicability distinguishes it from models such as Nelson-Siegel-Svensson, where a lack of high-quality data for many except the very largest economies interferes with model estimation.

The CIR process has three desirable properties. One, it produces rate paths that display autocorrelation, hence replicating the persistence of interest rates as observed in the empirical data. Two, the process obeys a lower bound. In the CIR, that lower bound is zero while, in practice, the lower bound might actually be slightly below zero at -0.5% or -0.75% due to the costs associated with holding cash. While a zero lower bound is a slight approximation, it suffices for CRA's purposes. Three, CIR processes will have a tendency to revert back to a long-run mean interest rate. That means simulated rate paths are also bounded from above. For rating assessment usage, the last property is relevant as we would not want to rely on a model where, in principle, interest-rate stresses can become arbitrarily large.

In continuous time, the CIR process can be written as follows:

$$dr_t = \kappa(\theta - r_t)dt + \sigma\sqrt{r_t}dW_t.$$

Here,  $r_t$  denotes the interest rate to be simulated. In CRA's typical applications, it corresponds to a short-term reference rate such as the three-month EURIBOR or the overnight SONIA.  $W_t$  denotes a Wiener process. The noise that enters through the increment  $dW_t$  is to be interpreted as unexplainable market risk. Parameter  $\theta > 0$  is the rate's long-run mean.  $\kappa > 0$  is a speed-of-adjustment parameter. The larger is  $\kappa$ , the stronger the drift back towards the long-run mean following a displacement of  $r_t$  through shocks represented by the last term on the right-hand side of the equation. If  $\kappa$  gets smaller instead, the rate becomes more persistent. The third and final model parameter  $\sigma$  influences the volatility of the interest rate: variability of  $r_t$  increases as  $\sigma$  rises.

<sup>2</sup> The CIR process was originally proposed in Cox, Ingersoll, and Ross, "A Theory of the Term Structure of Interest Rates," *Econometrica* 53, 387-407, March 1985.

Note how the square root of  $r_t$  is part of the right-hand side's last term. This square root goes to zero as  $r_t$  goes to zero. That is what ensures that the rate will not get negative in conjunction with a technical constraint on the size of  $\kappa$  being sufficiently large relative to the other parameters.<sup>3</sup>

To actually put the CIR to work in our quantitative cash flow models, the above equation needs to be discretized, as our models are set up in discrete time (typically, at monthly, quarterly, semi-annual, or annual frequencies). Applying the Euler-Maruyama discretization to the CIR ordinary differential equation above yields the following difference equation:

$$r_{t+\Delta t} - r_t = \kappa(\theta - r_t)\Delta t + \sigma\sqrt{r_t}\Delta t \varepsilon_t,$$

where  $\Delta t$  is the length of a period of time (e.g., a month) and  $\varepsilon_t$  is an error term that is i.i.d.  $N(0, 1)$ .

To simulate interest-rate movements, one may iterate on the equation representing the discretized CIR from above. In order to do that, one needs to fix the values for the parameters  $\kappa$ ,  $\theta$ , and  $\sigma$ . Additionally, an initial condition  $r_0$  representing the current interest rate is also required.

The discretized CIR also forms the basis for an estimation of the three model parameters. CRA relies on a calibration of model parameters in which both qualitative and quantitative factors are taken into account. This is discussed in more detail in the following section. However, at this point, it shall be noted that estimation of the equation above via maximum likelihood (for various interest rates) is part of our quantitative analysis. As the equation is univariate, the only data required for estimation is a historical time series of the interest rate under consideration.

### 3 CIR Model Parameterization

CRA's approach to CIR process parameterization is based on both quantitative as well as qualitative considerations. In this section, we provide an overview of our approach and disclose our calibration for three interest rates of global importance.

CRA considers an approach based solely on empirical estimation insufficient for several reasons. For one, interest rate time series often exhibit heteroskedasticity, alternating between periods of high persistence with low volatility and periods of lower persistence accompanied by elevated volatility. Thus, parameter estimates may vary with the sample being used. For another, we require a CIR specification for a large number of countries and currency areas. In some of these, the data quality and applicability are limiting factors. For instance, in estimations using the three-month EURIBOR, several years in the second half of the 2010s would have to be excluded from the sample due to rates being negative at the

<sup>3</sup> This technical condition will be fulfilled by the model parameterizations chosen by CRA.

time.<sup>4</sup> Furthermore, structural breaks—such as changes in monetary policy regimes—may also limit the usability of a historical time series.

CRA therefore calibrates the CIR model using a combination of internal estimates, external research, and qualitative considerations. Where a given calibration is subject to material uncertainty, CRA may conduct sensitivity analyses to assess the impact of alternative parameter assumptions.

Where a transaction is exposed to different tenors of the same interest rate (e.g., three-month as well as six-month EURIBOR), CRA typically does not calibrate the model separately for each tenor, provided that no tenor exceeds twelve months. However, given the flexibility of the CIR framework to accommodate specific tenors, analysts may deviate from this standard approach where transaction-specific characteristics warrant it.

CRA's calibration of the long-run mean of the interest rate  $\theta$  centers around the Fisher equation according to which the long-run/equilibrium nominal rate approximately equals the sum of the equilibrium real rate (typically denoted by  $r^*$ ) and the long-run expected inflation rate (typically denoted by  $\pi^*$ ). For most countries and currency areas relevant to CRA's ratings, the monetary policy regime can be considered stabilizing, and an explicit inflation target exists. We therefore set  $\pi^*$  equal to the respective inflation target. Recently, there has been considerable research on the magnitude of  $r^*$  for several major economies. Based on such studies, we pin down  $r^*$  to a value in line with consensus estimates. Once we have  $\pi^*$  and  $r^*$ , the value of  $\theta$  is determined as their sum by virtue of the Fisher equation.

For every interest rate under consideration, CRA sets the values of the speed-of-adjustment parameter  $\kappa$  and that of the instantaneous volatility  $\sigma$  jointly, as both will impact the volatility of the CIR-simulated interest rate series  $r_t$ . For global interest rates, our own current estimates of  $\kappa$  roughly fall in the range of 0.02-0.1 for quarterly data. For  $\sigma$ , estimates around 0.05-0.06 are common. We calibrate these values informed by both our internal estimates as well as publicly available ones, taking into account data-quality issues (if any).

CRA calibrates the initial condition  $r_0$  using the most recent available observation of the relevant interest rate, typically based on the latest monthly average.

For illustrative purposes, suppose we specified a quarterly calibration of the CIR with  $\theta = 4\%$ ,  $\kappa = 8\%$ , and an initial condition  $r_0 = 5\%$ . Under these assumptions, the half-life of an initial one-percent rate

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<sup>4</sup> As rates cannot get negative in the CIR model, the likelihood of negative rates is not well-defined. While the literature proposes extensions to the CIR so that negative rates can be handled, the extended models are more complex and do not fully meet our requirement of a simple and tractable model.

perturbation is around 8 1/2 years. Setting  $\kappa = 15\%$  instead would reduce the half-life to approx. 4 2/3 years.

Table 1 below shows our calibration of the CIR process parameters (targeting a quarterly simulation) for three global interest rates. Note that initial conditions  $r_0$  are not provided as we update these on a monthly basis.

Table 1: CIR Model Calibration for Select Interest Rates (For Tenors up to One Year) | Source: CRA

Parameter	EURIBOR	T-Bill	SONIA
$\theta$	0.02	0.03	0.025
$\kappa$	0.08	0.08	0.08
$\sigma$	0.06	0.06	0.06

CRA maintains parameterizations of the CIR corresponding to the central reference rates of other major developed countries. For brevity, these are not shown here.<sup>5</sup>

## 4 Integration of Simulated Rates into Cash Flow Models

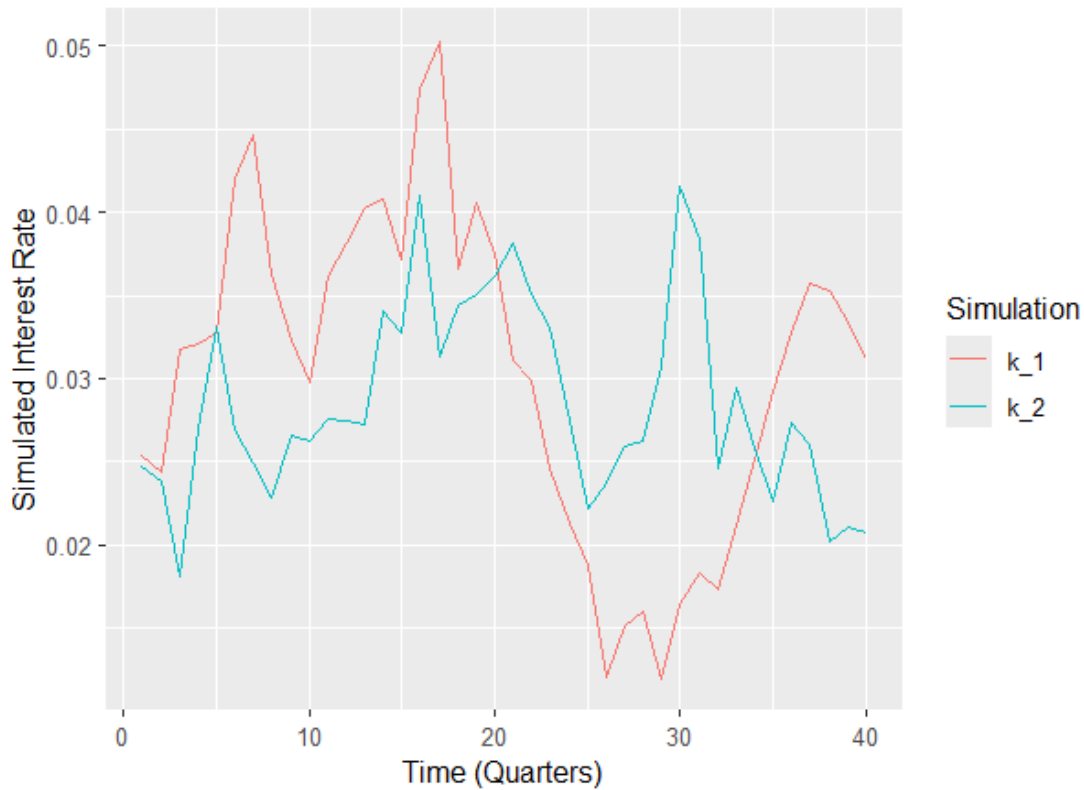
While CRA rates a wide variety of asset classes, from the perspective of integrating interest-rate scenarios into our quantitative cash flow models, a coarse distinction between two classes of models suffices. When rating transactions involve highly granular and homogeneous asset pools, CRA typically relies on “deterministic” cash flow models in which fixed base-case parameters (that often represent an expected value subject to a haircut or stress) enter. For less granular asset pools, CRA typically employs a Monte Carlo (MC) simulation-based cash flow modeling approach.

The CIR processes (calibrated as outlined in the previous section) integrate naturally into MC simulations. For each Monte Carlo trial and for each relevant interest rate, CRA applies the discretized CIR model to simulate an interest rate path over the transaction’s remaining term. These paths are then integrated into the cash flow model alongside other stochastic components. A common use case in CRA’s analytical work is that the CIR-generated interest rates that enter the model are reference rates such as SONIA. Often times, these are passed into the cash flow model as base rates which must be properly combined with cash or PIK margins to accurately reflect deal-specific properties.

<sup>5</sup> If interest-rate exposures relate to a country for which we do not have a CIR model calibration in place, we will use the following approximation. In downward interest-rate scenarios, we will assume an interest rate of zero. In upward scenarios, we will assume it equals the maximum observed historically (over a sample analysts consider appropriate). It shall be pointed out this approximation approach typically comes into play when an exposure to a certain rate is of secondary importance (e.g., one portfolio asset accounting for merely 1% of total portfolio NAV pays interest in reference to a base rate for which we have no calibration in place).

As an illustrative example, Figure 1 displays two paths corresponding to a hypothetical reference rate (not further specified). These would be used for two out of a large number of Monte Carlo trials (all of which aid the determination of a quantitative rating).

Figure 1: CIR-Generated Interest-Rate Paths

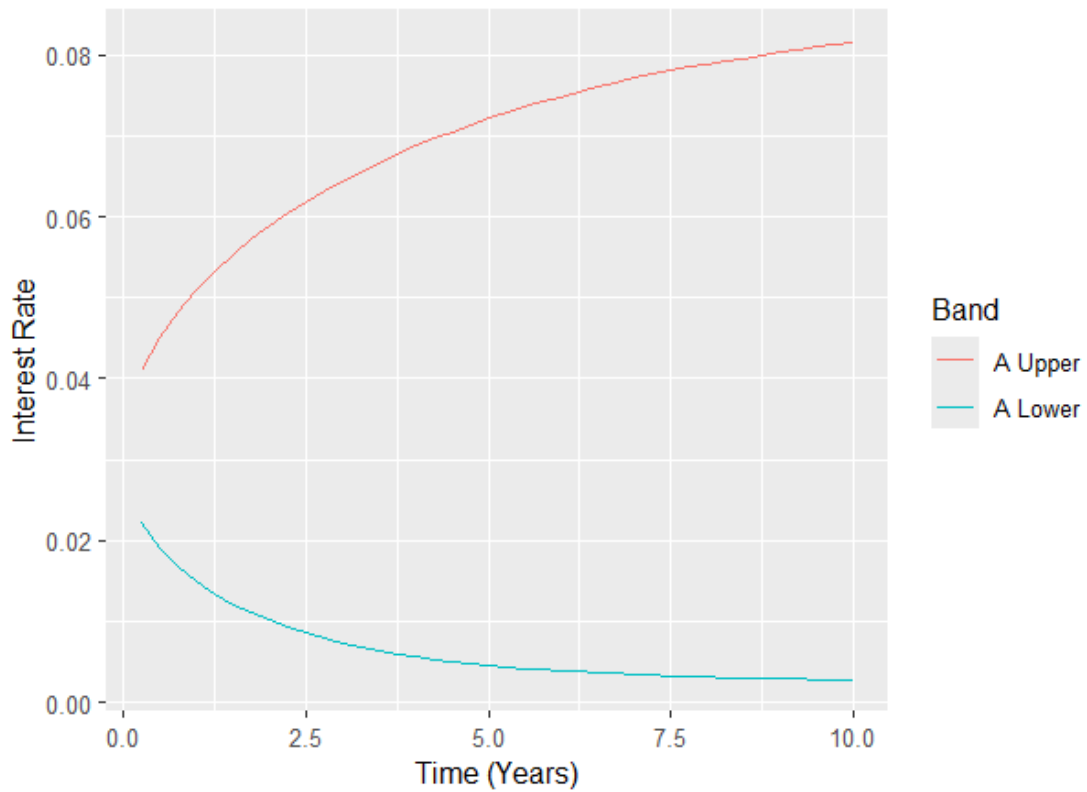


In the case of deterministic cash flow models, which CRA typically applies to highly granular and homogeneous portfolios—such as those commonly found in ABS transactions—it is not operationally feasible to generate and evaluate many different interest rate paths directly within the model. This is due to the design of such models, which rely on deterministic input parameters defined for specific rating scenarios.

Therefore, CRA has developed an alternative approach to deriving interest rate scenarios well-suited for deterministic frameworks. For each rating scenario, two stress bands are constructed: an upper band representing a rising interest rate environment and a lower band representing a falling rate scenario. These stress bands are derived from a large number of simulations of the CIR process, from which specific percentiles are selected to represent rating-dependent stress levels.

It should be noted that the stress bands are dynamic and adjust over time in response to changes in the prevailing interest rate level. Figure 2 illustrates the upper and lower stress bands corresponding to a selected rating category.

Figure 2: CIR-Generated Stress Bands



As shown in Figure 2, the stress bands are asymmetric. This reflects the inherent skewness of the CIR-generated interest rate distributions, which result from the model's lower bound constraint. Where a transaction is exposed to both rising and falling interest rate scenarios, CRA conducts separate analyses using the respective upper and lower stress bands. If the exposure is limited to one directional risk—either rising or falling rates—CRA applies only the relevant stress band in its analysis.